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Small Yukawa Couplings from Type I String Theory and the Inflationary Solution to the Strong CP and μ Problems

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Abstract

We investigate the origin of phenomenologically interesting small Yukawa couplings in Type I string theory. Utilising the framework of intersecting sets of D9 and orthogonal D5 branes we demonstrate the connection between extra dimensional volumes and Yukawa couplings. For example, we show that extra dimensions with inverse lengths of 10^8 GeV can lead to 10^{-10} Yukawa couplings. String selection rules, arising from the D-Brane setup, impose non-trivial constraints on the set of allowed superpotentials. As a phenomenological application of these results we construct a type I string model of inflationary particle physics which involves small Yukawa couplings of order 10^{-10}, and simultaneously solves the strong CP and μ problem of the MSSM, via the vacuum expectation value of the inflaton field.

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1 Introduction

Small Yukawa couplings are ubiquitous in the the flavour sector of the Standard Model describing the light fermion masses arising from the Higgs mechanism. An extreme example of such small Yukawa couplings is provided by Dirac neutrinos which would require Yukawa couplings of order 10^{-12} in order to account for the very small masses. Small Yukawa couplings also arise in other theories beyond the Standard Model, for example some theories of inflation where the inflaton potential is required to be very flat. The purpose of this paper is to discuss the origin of small Yukawa couplings in the framework of type I string theory, and to describe an application of the results to an inflationary particle physics model which involves small Yukawa couplings of order 10^{-10}, and simultaneously solves the strong CP and μ problem of the MSSM, via the vacuum expectation value of the inflaton field.

Type I string theory typically also involves the notion of intersecting Dirichlet-branes (D-branes) [1, 2]. The study of such frameworks has provided both a powerful model-building tool and an elegant intuitive picture of the resulting "brane worlds". The particles seen in nature are the lowest energy excitations of strings stretching between branes. They transform as chiral representations of the gauge groups that correspond to the coincident stacks of D-branes. The stacks of D-branes considered in this paper are both D9-branes, that fill spacetime, and three sets of D5-branes, that wrap 2-tori. The D5-brane stack's intersection is Minkowski space and they are orthogonal in the extra dimensions. When the spatial dimensions are reduced to four, by orbifold compactification, the gauge and Yukawa couplings of each set of branes become functions of the dimensions they wrap [3]. It is the connection between dimensions and couplings that we investigate in this paper. *The crucial point for model building is the fact that small Yukawa couplings are readily achievable in this framework.* We discuss the conditions under which these small couplings can

arise and construct a phenomenologically viable model utilising these couplings. The model in question was recently proposed in [4] as a field theory model involving small Yukawa couplings of order 10^{-10} and provides an inflationary solution to the μ and strong CP problems. Our main focus here is in the string construction of the model, and in particular on the origin of the small Yukawa coupling present in the model.

The outline of the rest of the paper is as follows. Section 2 examines the origins of the string superpotential and the couplings in the theory. We review the field theory model in section 3. Then, in subsection 3.2, we demonstrate how the model could be embedded in string theory. Subsection 3.3 introduces the string treatment of the soft terms and provides motivation for their spectrum. In subsection 3.4 we use new formulae for the soft terms in the presence of twisted moduli and show that there exist solutions consistent with the requirements of our model. Section 4 concludes the paper.

2 Couplings and Dilaton/Moduli

We will now review the properties of Type I string theory relevant for model building, first presented in [3]. We will be working with a D-brane setup which includes a geometric mechanism for generating small gauge and Yukawa couplings. We consider the class of spaces known as orientifolds (see [5] for a study of possible orientifolds) requiring the addition of intersecting stacks of orthogonal D5-branes and space-filling D9-branes for consistency. These spaces are all constructed from a 6-torus and it is the volume and anisotropy of this torus that leads to the generation of a hierarchy of couplings. The 6-torus itself is constructed out of three 2-tori each of which has one radius associated with it. We will show that if one radius is of order 10^{-8} GeV^{-1} and the other two are of order 10^{-18} GeV^{-1} then we obtain a coupling of order 10^{-10}.

After compactification we end up with, in the most general case, a model consisting

of three orthogonal stacks of D5-branes and a stack of D9-branes. Each D5-brane wraps around a 2-torus and its gauge coupling depends on the radius of the torus, via the moduli, T_i. The D9-branes wrap all the tori and so depend on all the radii, via the four dimensional dilaton, S. The moduli and dilaton take the following forms:

$$T_i = \frac{2R_i^2 M_*^2}{\lambda_I} + i\eta_i \tag{1}$$

$$S = \frac{2R_1^2 R_2^2 R_3^2 M_*^6}{\lambda_I} + i\theta \tag{2}$$

where η_i and θ are untwisted Ramond-Ramond closed string states, M_* is the string scale and λ_I is the ten dimensional dilaton which governs the strength of string interactions.

The gauge couplings on the branes can be determined from the S and T_i fields by

$$g_{5_i}^2 = \frac{4\pi}{Re(T_i)} \tag{3}$$

$$g_9^2 = \frac{4\pi}{Re(S)} \tag{4}$$

The non-canonical, $D = 4$, $\mathcal{N} = 1$ effective superpotential has only $\mathcal{O}(1)$ Yukawa couplings [6], but the Kahler metric, although diagonal, is significantly different from the identity. To understand our theory in the low energy, after the dilaton and moduli have acquired VEVs, we must canonically normalise the Kahler potential and take the flat limit in which $M_p \to \infty$ while $m_{3/2}$ is kept constant [7, 8]. This gives a theory containing superfields with canonical kinetic terms interacting via renormalisable operators. Notice that the Yukawa couplings can be identified with the gauge couplings (up to the $\mathcal{O}(1)$ factors present before normalisation):

$$\begin{aligned} W = g_9 \left(C_1^9 C_2^9 C_3^9 + C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1} + \sum_{i=1}^{3} C_i^9 C^{95_i} C^{95_i} \right) + \sum_{i,j,k=1}^{3} g_{5_i} \left(C_1^{5_i} C_2^{5_i} C_3^{5_i} \right. \\ \left. + C_i^{5_i} C^{95_i} C^{95_i} + d_{ijk} C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k} + \frac{1}{2} d_{ijk} C^{5_j 5_k} C^{95_j} C^{95_k} \right). \end{aligned} \tag{5}$$

where the C terms are low energy excitations of strings: charged chiral superfields. The superscripts denote the branes which the strings end on and terms with different subscripts transform differently under the gauge group associated with the brane. Appendix A.1 discusses these fields in more detail.

Associated with this superpotential there are a set of allowed soft breaking terms given in

$$\begin{aligned} V_{soft} = g_9 \Big(& A_{C_1^9 C_2^9 C_3^9} C_1^9 C_2^9 C_3^9 + A_{C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1}} C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1} \\ & + \sum_{i=1}^{3} A_{C_i^9 C^{95_i} C^{95_i}} C_i^9 C^{95_i} C^{95_i} \Big) + \sum_{i,j,k=1}^{3} g_{5_i} \Big(A_{C_1^{5_i} C_2^{5_i} C_3^{5_i}} C_1^{5_i} C_2^{5_i} C_3^{5_i} \\ & + A_{C_i^{5_i} C^{95_i} C^{95_i}} C_i^{5_i} C^{95_i} C^{95_i} + A_{C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k}} d_{ijk} C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k} \\ & + \frac{1}{2} d_{ijk} \, A_{C^{5_j 5_k} C^{95_j} C^{95_k}} C^{5_j 5_k} C^{95_j} C^{95_k} \Big) \\ & + \sum_{i=1}^{3} m_{C_i^9}^2 |C_i^9|^2 + \sum_{i=1}^{3} \Big(m_{C_1^{5_i}}^2 |C_1^{5_i}|^2 + m_{C_2^{5_i}}^2 |C_2^{5_i}|^2 + m_{C_3^{5_i}}^2 |C_3^{5_i}|^2 \Big) \\ & + \frac{1}{2} \sum_{i,j=1}^{3} d_{ij} m_{C^{5_i 5_j}}^2 |C^{5_i 5_j}|^2 + \sum_{i}^{3} m_{C^{95_i}}^2 |C^{95_i}|^2 \end{aligned} \tag{6}$$

where, in a slight abuse of notation, we have used the same notation for the superfields and their scalar components. Also we have defined $d_{ijk} = |\epsilon_{ijk}|$ and $d_{ij} = |\epsilon_{ij}|$.

The $D = 4$ Planck scale is related to the string scale by

$$M_p^2 = \frac{8 M_*^8 R_1^2 R_2^2 R_3^2}{\lambda_I^2}. \tag{7}$$

From this and Eqs. (2) to (4) we find that

$$g_{5_1} g_{5_2} g_{5_3} g_9 = 32\pi^2 \left(\frac{M_*}{M_p} \right)^2. \tag{8}$$

In the phenomenological application discussed in the next section, we shall require at least one coupling of $\mathcal{O}(10^{-10})$ and one of $\mathcal{O}(1)$. According to the above results, this constrains the size of our radii and the value of the string scale. For definiteness we consider the case where $g_{5_1} \sim 10^{-10}$ and the remaining gauge couplings are all

$\mathcal{O}(1)$. From Eq. (8) we see this is clearly allowed if we have a 10^{13} GeV string scale. Specifically our couplings are $g_{5_2} = \sqrt{\frac{4\pi}{24}}$ (to give $\alpha_{\mathrm{GUT}} = 1/24$, consistent with gauge coupling unification), $g_{5_3} = g_9 = 2$ and $g_{5_1} = 10^{-10}$ gives $M_* = 10^{13}$ GeV.

The hierarchy in gauge couplings corresponds to a hierarchy in the radii. Using Eqs. (1) and (3) for the above couplings we find that

$$R_1^{-1} = 1.3 \times 10^8 \text{ GeV} \tag{9}$$

$$R_2^{-1} = 9.1 \times 10^{17} \text{ GeV} \tag{10}$$

$$R_3^{-1} = 2.4 \times 10^{18} \text{ GeV.} \tag{11}$$

These radii are all too small to have Kaluza-Klein (KK) or winding modes that will be readily excitable at collider energies. The winding modes of R_1 are $\approx n10^{18}$ GeV and R_2 and R_3 have winding modes of $\approx n10^8$ GeV. The KK modes for R_1 are $\approx n10^8$ GeV and R_2 and R_3 are $\approx n10^{18}$ GeV. In principle these massive modes could affect inflation. However the inflationary scale is 10^8 GeV so it is unlikely that these modes would appear with any great abundance.

There is a caveat to Eq. (8). It is only valid when the untwisted moduli provide the dominant contribution to the gauge couplings. This will be true in the application discussed in the next section, to a reasonable approximation. For large (negative) values for δ_{GS} (see [9]) we need large twisted moduli (to be discussed in Section 3.4) on the Standard Model brane, $D5_2$, so the twisted modulus has a significant contribution to g_{5_2}. In this case it is better to consider the alternative formulation of Eq. (8) in terms of S/T_i:

$$\left(\frac{M_*}{M_p}\right)^2 = \frac{1}{2\big(Re(T_1)Re(T_2)Re(T_3)Re(S)\big)^{1/2}} \tag{12}$$

From this we can see that M_* only has a very weak dependence on $Re(T_2)$, $M_* \propto \big(Re(T_2)\big)^{-1/4}$. As such doubling $Re(T_2)$ only amounts to a 20% correction to M_*. Since the exact value of M_* is not crucial to our results we use the approximate

formula Eq. (8) exclusively in this paper.

It is worth noting that we could have chosen any of the three sets of five branes to have the tiny coupling, but D9 would not have been a good choice because of the following relation[1].

$$\lambda_I = \frac{g_{5_1} g_{5_2} g_{5_3}}{2\pi g_9} \tag{13}$$

To remain in the perturbative regime we require that λ_I be less than one. Clearly, swapping between ($g_9 \sim 10^{-10}$, $g_{5_1} \sim 1$) and ($g_9 \sim 1$, $g_{5_1} \sim 10^{-10}$) dramatically changes λ_I, assuming all the other couplings are left unchanged. The gauge coupling choices shown below Eq. (8) give $\lambda_I \sim 10^{-11}$ which is well inside the perturbative regime. It should be noted that, under T-Duality, these two cases are equivalent. However, if we want to perform string theoretic calculations a perturbative coupling is undeniably useful.

3 A Phenomenological Application

3.1 The Inflationary Solution to the Strong CP and μ Problems

As a phenomenological application of these results we shall discuss the supersymmetric field theory model proposed in [4] where small Yukawa couplings λ, κ of order 10^{-10} were invoked. To make this paper self-contained we briefly review the field theory model, are we refer the reader to [4] for a more detailed discussion. The idea of the model was to provide a simultaneous solution to both the μ problem and the strong CP problem, as well as providing a satisfactory model of hybrid inflation. Indeed the role of the inflaton ϕ is rather special in this model, since it not only provide inflation, but also its vacuum expectation value after inflation is directly responsible for the μ

[1] Again there is an alternative formulation in terms of the moduli and dilaton: $\lambda_I^2 = 2Re(S)/\big(Re(T_1)Re(T_2)Re(T_3)\big)$

term of the MSSM.

The starting point of the field theory model is the following superpotential and potential:

$$W = \lambda\phi H_u H_d + \kappa\phi N^2 + y_t Q H_u U + y_b Q H_d D. \tag{14}$$

$$\begin{aligned} V_{soft} =& \lambda A_\lambda \phi H_u H_d + \kappa A_\kappa \phi N^2 + y_t A_{y_t} Q H_u U + y_b A_{y_b} Q H_d D + h.c. \\ &+ m_0^2(|H_u|^2 + |H_d|^2 + |N|^2 + |U|^2 + |D|^2) + m_Q^2|Q|^2 + m_\phi^2|\phi|^2 \end{aligned} \tag{15}$$

ϕ and N are, respectively, the inflaton and waterfall fields (for a more detailed discussion of the inflationary epoch see [4] and the references therein). These fields are singlets of the Minimal Supersymmetric Standard Model (MSSM) [10] gauge group and the other fields are just the usual quarks and Higgs multiplets of the MSSM with standard MSSM quantum numbers as shown in table 1.

Fields	$SU(3)$	$SU(2)_L$	$U(1)_Y$
Q	3	2	1/6
U	$\bar{3}$	1	−2/3
D	$\bar{3}$	1	1/3
H_u	1	2	1/2
H_d	1	2	−1/2

Table 1: MSSM charges

This is a model of hybrid inflation [11, 12, 13, 14, 15] that simultaneously solves the μ [10] and strong CP problems. Hybrid inflation is a two field inflationary model with one field, the inflaton, that drives inflation and another field, the waterfall field, that ends inflation. The coupling of these two fields provides a field-dependent mass for the waterfall field which becomes tachyonic at a critical value of the inflaton, triggering the end of inflation. After inflation has ended the VEV of the inflaton $<\phi>$ both generates the μ term (in a similar way to the NMSSM [16, 17]), when $\lambda\phi H_u H_d \to \mu H_u H_d$ and breaks Peccei-Quinn (PQ) [18] symmetry solving the strong

CP problem. The PQ breaking scale was shown to be $< \phi >\sim 10^{13}$ GeV, and $\lambda \sim 10^{-10}$ then results in $\mu \sim 10^3$ GeV. Inflationary considerations also require $\kappa \sim 10^{-10}$.

This model is similar to the BGK model [19], but with the inflaton providing the μ term rather than the N field. In [4] it was assumed *ad hoc* that $\lambda = \kappa$ and $A_\lambda = A_\kappa$, where both of these results will shortly be derived from the string construction. This led to the prediction [4]:

$$8m_0^2 > |A_\lambda|^2 > 4m_0^2 \tag{16}$$

and hence:

$$\mu^2 = (0.25 - 0.5)m_0^2, \tag{17}$$

where m_0 is a universal soft scalar mass of order a TeV, whose universality will also be shown to result from the string construction. The relative smallness of the inflaton soft scalar mass, which is assumed to be of order an MeV, will also be discussed.

3.2 String Embedding of the Model

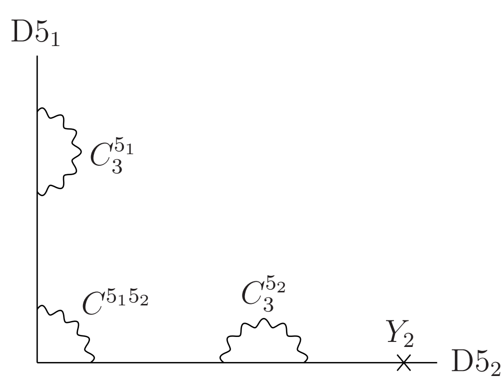

Figure 1: Schematic representation of two stacks of D5-branes. The stacks of branes overlap in Minkowski space, but are orthogonal in the compactified dimensions. The C states correspond to chiral matter fields and Y_2 is a twisted modulus (introduced in Section 3.4) localised within the extra dimensions, but free to move in Minkowski space. We have only presented the string states involved in our construction: for a more complete picture see Figure 1 in [9].

Now that we have the superpotential Eq. (5) and the Yukawa relationship, Eq. (8), we have all the tools necessary for the string construction. Figure 1 displays the two branes that feature in our construction. The $D5_2$ brane is assumed to have an order one gauge coupling and possesses a twisted moduli on the fixed point that is spatially separated from the $D5_1$ brane. All the MSSM fields transform under representations of the $D5_2$ brane's gauge group. The $D5_1$ brane is assumed to have an order 10^{-10} gauge coupling and hence no MSSM fields transform under it. This corresponds to the choice of gauge couplings already discussed in section 2.

The specific string assignments we assume are given in table 2.

ϕ	N	Q	H_u	H_d	U	D
$C_3^{5_1}$	$C^{5_1 5_2}$	$C_3^{5_2}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$

Table 2: String Assignments

g_{5_1}	$C_3^{5_1}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$	g_{5_2}	$C_3^{5_2}$	$C^{5_2 5_1}$	$C^{5_2 5_1}$
λ	ϕ	N	N	y_t	Q	H_u	U
κ	ϕ	H_u	H_d	y_b	Q	H_d	D

Table 3: String allowed Yukawa couplings arising from the string assignments in table 2.

The allowed couplings which result from the assumed string assignments follow from the string selection rules embodied in the superpotential in Eq. (5), and are written explicitly in table 3. Table 3 demonstrates that, with the assumed string assignments, we can reproduce the field theory model superpotential in Eq. (14). We emphasise that we now have a stringy explanation for the small Yukawa couplings in the field theory model, since in the string construction they are equal to the small gauge couplings $g_{5_1} \sim 10^{-10}$ whose smallness has a string origin as discussed in section 2. Note that the allowed couplings in table 3 are precisely the ones assumed in the superpotential, Eq. (5), and gauge invariance means that the only terms we are allowed to write down are those found in Eq. (14), and no others. Furthermore the

string selection rules now require that $\lambda = \kappa = g_{5_1} \sim 10^{-10}$, and $y_t = y_b = g_{5_2} \sim 1$, previously assumed in an *ad hoc* manner in the field theory version of the model.

Table 3 also makes clear that $g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}$ and $g_{5_2} C_3^{5_2} C^{5_2 5_1} C^{5_2 5_1}$ cannot produce any further gauge invariant superpotential terms. Also, since we have only assigned fields to $C_3^{5_1}$, $C_3^{5_2}$ and $C^{5_1 5_2}$ the only terms in Eq. (5) that we can use are the two superpotential terms that appear in table 3. It should be highlighted that the assignment of quark Yukawas is non-trivial if there is a direct connection (i.e. a renormalisable coupling) between the inflationary interactions and MSSM fields. For example, if the μ term were provided by the N field, as in [19], it would not be possible to write down quark Yukawas consistent with the inflationary sector of the model.

The methodology underlying the selection of these assignments is the subject of a detailed discussion in Appendix A.

3.3 Soft Terms

Given the assumption that the SUSY breaking is dominated by the dilaton/moduli sector we can parametrise its effect in terms of $m_{3/2}^2$ and Goldstino angles (that parametrise the contributions from different sources of SUSY breaking). The following, Goldstino angle independent, sum rule is crucial to our model as we will soon see:

$$m^2_{C_3^{5_1}} + 2m^2_{C^{5_1 5_2}} = |M_{5_1}|^2 = |A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}}|^2 \tag{18}$$

In order to obtain successful slow roll [20] we need to have a small soft mass for the inflaton, ϕ, (MeV or less), which presents us with a potential difficulty. All the soft mass squareds coming from the dilaton/moduli SUSY breaking are, assuming vanishing cosmological constant, of the form $m_{3/2}^2 F(\theta, \Theta_i)$ where F is some, typically order one, function of the Goldstino angles, θ, Θ_i. If we appeal to Gaugino condensation we

can motivate the selection of Goldstino angles resulting in a small mass for ϕ since this allows explicit calculation of the soft terms. In [21] the soft spectrum resulting from gaugino condensation was shown to include $m^2_{C_3^{5_1}} \approx 0$ and $m^2_{C^{5_1 5_2}} \approx \frac{1}{2} m^2_{3/2}$. From (18) we see that $|A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}}|^2 \approx m^2_{3/2}$. The validity of the sum rule is noted in their paper and they present a consistent, explicit expression for the trilinear couplings.

It is worth mentioning that this is true for orbifold compactifications in the absence of twisted moduli, but not for more general spaces, such as Calabi-Yau manifolds [22], but for the purposes of this paper we restrict ourselves to orbifolds.

Now we must consider the implications of this sum rule for our model. From Eq. (18) we have an explicit relationship between the soft masses and trilinears for the relevant string assignments. From table 2 we know that ϕ is assigned to $C_3^{5_1}$ and that N is assigned to $C^{5_1 5_2}$. Since $m_\phi \approx 0$ and N, H_u and H_d have the same string assignment Eq. (18) becomes

$$2m_0^2 \approx |A_\lambda|^2, \tag{19}$$

where the string prediction is that $A_\lambda = A_\kappa = A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}}$, and $m_0^2 = m^2_{C^{5_1 5_2}}$, where we have assumed that $m_\phi^2 = m^2_{C_3^{5_1}} \approx 0$. Note that in [4] it was assumed *ad hoc* that $A_\lambda = A_\kappa$, but now this result follows immediately from the string construction.

Unfortunately Eq. (19) does not satisfy the lower bound on the trilinears shown in Eq. (16) so the standard soft terms are inconsistent with our model. Since the soft terms depend, via the SUGRA potential, on the F-terms and the Kahler potential [8] we must look to modifying these. In fact in type I string theory it is known that there are in general twisted moduli which, if their F-terms have non-zero VEVs will lead to additional contributions to supersymmetry breaking described by extra Goldstino angles [9], which will lead to a violation of the above sum rule for soft masses. In the following section we consider the Kahler potential in the presence of one twisted modulus located at a fixed point in the 2-torus wrapped by D5_2.

3.4 Soft terms with Twisted Moduli

We want to quantify the effects of twisted moduli on our string set up and see if it is possible to include them in such a way as to give acceptable phenomenology. We follow the analysis in [9], but generalise it to allow $\lambda_I \neq 1$. Our work will show that this produces soft terms consistent with those presented in [9] when $\lambda_I = 1$, but that λ_I has a significant effect on the soft spectrum. We consider the case of just one twisted modulus, Y_2, living on D5_2 at the fixed point shown on figure 1. Leaving the derivation of the modified soft terms to Appendix B, we quote the important results here. First the soft masses:

$$\begin{aligned} m_Q^2 = m^2_{C_3^{5_2}} &= m_{3/2}^2 - 3m_{3/2}^2\Theta_1^2\cos^2\theta\sin^2\phi \\ m_0^2 = m^2_{C^{5_1 5_2}} &= \tilde{m}^2 - \frac{3}{2}m_{3/2}^2\left(\sin^2\theta + \Theta_3^2\cos^2\theta\sin^2\phi\right) \\ m_\phi^2 = m^2_{C_3^{5_1}} &= \tilde{m}^2 - \frac{3}{k}m_{3/2}^2\Theta_2^2\cos^2\theta\sin^2\phi. \end{aligned} \tag{20}$$

where Goldstino angles θ and ϕ and Goldstino parameters Θ_i (where $\sum_{i=1}^3 \Theta_i^2 = 1$) parametrise the supersymmetry breaking as discussed in [9]. $\tilde{m}$ contains all the λ_I dependence and

$$k = 1 + \delta_{GS}\left(Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right). \tag{21}$$

In full generality $\tilde{m}$ is an unwieldy expression, to be found in Appendix B, but when $\lambda_I \ll 1$, it simplifies considerably. To very good approximation $\tilde{m} \approx m_{3/2}$ in our model. The full expression for the trilinear associated with the inflationary superpotential terms is quoted in Appendix B. The approximate result when $\lambda_I = 10^{-11}$ is

$$\begin{aligned} A_\lambda = A_{C_3^{5_1}C^{5_1 5_2}C^{5_1 5_2}} \approx -\sqrt{3}m_{3/2}\cos\theta\Big[&\sin\phi\,\Theta_1\,e^{i\alpha_1} \\ &-\cos\phi\,e^{i\alpha_{Y_2}}\left\{Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right\}\Big] \end{aligned} \tag{22}$$

We now need to analyse these expressions to see if it is possible to satisfy the bounds laid out in section 3. In this paper we do not explicitly calculate the soft terms, but

rely on [21] to motivate the plausibility of $m_\phi^2 \sim 0$. For our purposes we simply wish to see if it is possible to satisfy the bounds for a particular choice of parameters. To see this we have performed a Monte Carlo analysis to generate a random set of Goldstino parameters in order to check that the masses and trilinears fit within the allowed ranges. In addition to the inflationary requirements we only accepted parameters that gave positive soft mass squareds for the quark doublet, Q.

Since $m_\phi^2 = 0$ can only be satisfied if k is within the following range

$$0 < k \leq 3 \tag{23}$$

the allowed values for $Y_2 + \overline{Y}_2$ and $T_2 + \overline{T}_2$ are restricted by Eq. (21). We also wish to retain gauge coupling unification which places further constraints on our moduli VEVs since we require that:

$$\frac{4\pi}{g_\alpha^2} = Re(f_\alpha) = Re(T_2) + \frac{s_\alpha}{4\pi} Re(Y_2) \approx 24 \tag{24}$$

where s_α are model dependent coefficients and α runs over the the different gauge groups. In some orientifolds s_α can be of the same order as the beta functions, as discussed in [3], but we make the simplifying assumption that they are all set to 4π. The other simplifying assumption we make is to set all the phases to 1.

The Monte Carlo routine starts with a particular value for δ_{GS}, generates a random set of Goldstino parameters, k and the moduli that satisfy gauge coupling unification and then calculates the required masses and trilinears. When this is complete it checks to see if the constraints on the masses are satisfied and if they are then the parameters are accepted.

Sample points are displayed in table 4 with soft masses in units of $m_{3/2}$. These sample points all satisfy the condition in Eq. (16), as required. Also from [4] we know that, in order to satisfy slow roll conditions, $m_\phi \sim$ MeV is required. In our approach this will require tuning of the Goldstino angles, to yield the MeV inflaton soft masses,

but as already mentioned this may arise dynamically in certain approaches [21]. Note that the Monte Carlo has accurately determined the Goldstino angles necessary to achieve such MeV soft masses explicitly, but in table 4 they have been rounded for brevity.

δ_{GS}	θ	ϕ	Θ_1	Θ_2	Θ_3	m_0^2	m_Q^2	A_λ	A_Q	$Re(T_2)$	$Re(Y_2)$
-2	5.69	5.33	0.807	0.173	0.565	0.312	0.113	1.34	1.40	27.9	-3.92
-2	5.54	5.25	0.810	0.497	0.311	0.248	0.215	1.11	1.23	27.8	-3.84
-4	6.14	5.20	0.463	0.176	0.869	0.100	0.507	0.887	1.19	32.2	-8.22
-4	5.57	5.26	0.777	0.407	0.479	0.219	0.237	1.01	1.13	32.2	-8.23
-6	5.54	4.74	0.705	0.529	0.472	0.13	0.195	0.901	1.00	36.9	-12.9
-6	5.39	3.88	0.937	0.163	0.31	0.0679	0.532	0.551	0.867	36.8	-12.8
-8	6.68	5.19	0.693	0.156	0.704	0.279	0.0313	1.07	1.09	41.6	-17.6
-8	7.18	4.57	0.696	0.707	0.127	0.0724	0.448	0.735	0.992	41.7	-17.7
-10	6.4	4.94	0.379	0.451	0.808	0.0625	0.596	0.652	1.02	46.7	-22.7
-10	5.43	4.34	0.916	0.316	0.248	0.117	0.0505	0.938	0.963	46.6	-22.6

Table 4: Goldstino parameters and soft terms satisfying Eq. (16)

4 Conclusions

We have discussed the possibility of generating small Yukawa couplings within the framework of Type I string theory, and emphasised the importance of this result for model building. We have seen that it is possible to obtain very small Yukawa couplings without requiring an exceptionally low string scale or especially large extra-dimensions. In the example discussed we obtained Yukawa couplings of order 10^{-10} with a string scale of order 10^{13} GeV, and the largest extra dimensions having a compactification scale of order 10^8 GeV.

As an application of these results we constructed a type I string theory realisation of the model presented in [4] which simultaneously solves the μ and strong CP problems while providing a viable description of hybrid inflation. The model provides a good example of the approach since it requires exceptionally small Yukawa couplings of order 10^{-10}, together with certain constraints on the soft mass parameters, which

from the field theory point of view looks quite unappealing and *ad hoc*. We have shown how the model can originate from a type I string theory setup consisting of intersecting D9 and D5-branes where the required small Yukawa couplings and soft mass relations can readily emerge. Twisted moduli play an important part in determining the soft spectrum, and we have provided a first discussion of their effects away from the $\lambda_I = 1$ limit.

The model discussed provides a convenient example in which small Yukawa couplings and soft mass relations can arise quite naturally from a type I string construction. It is clear that the type I string approach discussed here is much more general than this specific model, and has very wide applicability. A possible further application is to Dirac neutrinos, which also involve very small Yukawa couplings, and we plan to discuss this in a dedicated future publication.

A Methodology

There are three main rules that must be adhered to when attempting to realise field theoretic models in the D-Brane framework. One: each superfield may only be assigned to one string state. Two: each string state can have several different superfields assigned to it. Three: all tree level superpotential terms have to be found in Eq. (5). Notice that the superpotential terms in Eq. (5) have one of two forms: either one field linear and the other quadratic (e.g. $C_k^{5_i} C^{5_i 5_j} C^{5_i 5_j}$), or all three fields linear (e.g. $C_1^{5_i} C_2^{5_i} C_3^{5_i}$). There are no cubic terms in the string superpotential.

The aim of this approach is to take a purely field theoretic superpotential and see if it can be realised in the string superpotential, using the rules as laid out. To see the terms allowed we want to consider all possibilities in turn. First let us categorise all possible renormalisable superpotential terms that we might want to realise in the string superpotential.

For simplicity consider a toy field theory with just three, gauge singlet, superfields A, B and C and the following superpotential.

$$W = aABC + bAAB + cAAA \tag{25}$$

Let us demonstrate the assignment of each term in Eq. (25) individually.

$aABC$ could be assigned to any term in Eq. (5), obviously a string term of the trilinear form $C_1^{5_i} C_2^{5_i} C_3^{5_i}$ would be acceptable, leading to different assignments for each superfield. For example A, B and C could be assigned to $C_1^{5_i}$, $C_2^{5_i}$ and $C_3^{5_i}$ respectively. A quadratic term like $C_i^9 C^{95_i} C^{95_i}$ could also be used since B and C can share the same assignment, C^{95_i}. For example A assigned to C_i^9 and both B and C assigned to C^{95_i}. Now AAB can only be assigned to quadratic terms: rule one forbids trilinear terms since we would be forced to assign one field to two string states. We can clearly see that AAA cannot be assigned to either quadratic or trilinear for the same reasons. Incidentally this forbids the NMSSM at tree level since it requires a cubic superfield.

We have now laid out the string selection rules underlying our construction. Obviously each term allowed by the string selection must be gauge invariant and have the correct coupling for it to appear in the theory. Once we have made all the necessary assignments we write down all terms allowed by gauge invariance and string selection. For example, even if c were set to zero, we could not get Eq. (25) exactly as we would be forced to include the term $bCCB$. With the superpotential written down and the Kahler potential canonically normalised the supersymmetric side of the construction is complete. The SUSY breaking terms are considered earlier in the text in sections 3.3 and 3.4. In the following section we explicitly construct our theory.

A.1 String Assignments

First we look for all possible superpotential terms containing a squared superfield and with the correct coupling, g_{5_1}, to accommodate $\kappa\phi N^2$. There are three terms that satisfy this requirement:

$$(i)\ g_{5_1} C_1^{5_1} C^{95_1} C^{95_1}, (j)\ g_{5_1} C_2^{5_1} C^{5_1 5_3} C^{5_1 5_3} \text{ and } (k)\ g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2} \tag{26}$$

(j) and (k) are symmetric under relabelling of 2 and 3. Notice that Eq. (5) is symmetric under permutations of the 1, 2 and 3 labels since you are free to choose your radii at the start of the construction. T-Duality and relabelling links all possible permutations of the branes hence (i), (j) and (k) are equivalent at this stage of the construction. Due to these symmetries we only consider the $C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}$ term. Since we know that ϕ is assigned to $C_3^{5_1}$ we look for the $\lambda\phi H_u H_d$ term.

There are only two terms that include ϕ: (a) $g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}$ and (b) $g_{5_1} C_1^{5_1} C_2^{5_1} C_3^{5_1}$. Notice these are inequivalent under T-Duality.

Now the possible gauge assignments become important. Superfields transform under gauge groups according the branes they (or rather the string which they are an excitation of) stretch between. For example a $C_3^{5_1}$ state corresponds to a string that starts and ends on the same set of branes, $D5_1$, and so can only have gauge quantum numbers from that stack. On the other hand a $C^{5_1 5_2}$ state ends on both the $D5_1$ branes and the $D5_2$ branes and so can have quantum numbers from both.

Naturally we must ensure that our fields transform appropriately under our symmetry group, in our case the MSSM's, and that each term is invariant. For term (b) each Higgs superfield must transform under the $D5_1$ brane's gauge group with gauge coupling $g_{5_1} \sim 10^{-10}$. Since we expect the standard model gauge couplings to be order one at the string scale this is unacceptable. This leaves (a): both Higgs can transform correctly since we only need one non-Abelian gauge factor, $SU(2)_L$, and

this can come from the D5_2 brane with its order one coupling. The final step is to see if $\mathcal{O}(1)$ quark Yukawas are consistent with these assignments.

With H_u and H_d both assigned to $C^{5_15_2}$, the order one terms we can write down are

$$(\alpha) g_{5_2} C_1^{5_2} C^{5_25_1} C^{5_25_1}, \; (\beta) g_9 C^{5_25_3} C^{5_15_2} C^{5_35_1} \text{ and } (\gamma) g_{5_3} C^{95_2} C^{5_15_2} C^{95_1} \tag{27}$$

To enforce gauge invariance the quark doublet must live on an assignment with ends on branes with $\mathcal{O}(1)$ gauge couplings. With this in mind the simplest choice is (α) since it has all the standard model gauge factors coming from one brane. The second and third choices necessitate diagonal symmetry breaking from $\big(SU(3) \times SU(2) \times U(1)\big)^2$ to $SU(3) \times SU(2) \times U(1)$. With this caveat all three solutions are valid models, but chose to focus on the simpler model without diagonal symmetry breaking in the rest of the analysis. This concludes the supersymmetric construction.

To compare these assignments with a T-Dual model we list the possible assignments and a T-Dual set in Appendix A.2.

A.2 Alternative assignments

The following table contains all allowed assignments once $\lambda\phi N^2$ is assigned to either $C_1^{5_1} C^{95_1} C^{95_1}$ or $C_3^{5_1} C^{5_15_2} C^{5_15_2}$. In the first block the couplings are $g_9 = \sqrt{\frac{4\pi}{24}}$, $g_{5_3} = g_{5_2} = 2$ and $g_{5_1} = 10^{-10}$ and the second block has g_9 and g_{5_2} exchanged. As pointed out in Appendix A.1 the two sets of assignments are equivalent. We only include them both to allow the reader to compare and contrast both cases.

B Twisted soft terms

To include the effects of twisted moduli the Kahler potential must be modified. The twisted moduli need a quadratic term in the Kahler potential to provide their kinetic

ϕ	N	Q	H_u	H_d	U	D
$C_1^{5_1}$	C^{95_1}	C_1^9	C^{95_1}	C^{95_1}	C^{95_1}	C^{95_1}
$C_1^{5_1}$	C^{95_1}	C^{95_2}	C^{95_1}	C^{95_1}	$C^{5_1 5_2}$	$C^{5_1 5_2}$
$C_1^{5_1}$	C^{95_1}	C^{95_3}	C^{95_1}	C^{95_1}	$C^{5_1 5_3}$	$C^{5_1 5_3}$
$C_3^{5_1}$	$C^{5_1 5_2}$	$C_3^{5_2}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$
$C_3^{5_1}$	$C^{5_1 5_2}$	$C^{5_2 5_3}$	$C^{5_1 5_2}$	$C^{5_1 5_2}$	$C^{5_3 5_1}$	$C^{5_3 5_1}$
$C_3^{5_1}$	$C^{5_1 5_2}$	C^{95_2}	$C^{5_1 5_2}$	$C^{5_1 5_2}$	C^{95_1}	C^{95_1}

Table 5: Collated assignments

terms:

$$\hat{K}(Y_2, \overline{Y}_2) = \frac{1}{2}\left[Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right]^2. \tag{28}$$

The $\delta_{GS}\ln(T_2 + \overline{T}_2)$ term describes anomaly cancellation via Green-Schwarz mixing [23].

To enforce the sequestration of states the Kahler potential must be modified accordingly. A multiplicative factor ξ must be introduced to every string state that does not overlap significantly with the twisted modulus. For example a $C_2^{5_2}$ state comes from a string which starts and ends on D5_2 so it interacts with Y_2 and feels no suppression. An example of a sequestered state is $C^{5_1 5_2}$: although one end of the string couples to D5_2 the other attaches to D5_1 and string tension localises the string at the origin, away from the fixed point occupied by Y_2.

These two modifications are expressed in the following equations:

$$\begin{aligned} K(S, \overline{S}, T_i, \overline{T}_i, Y_2, \overline{Y}_2)_{seq.} = & \frac{1}{2}\left[Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right]^2 \\ & + \sum_{i\neq 2} \frac{\xi(T_2, Y_2)}{(S+\overline{S})}|C_i^{5_i}|^2 + \frac{1}{2}\sum_{i\neq 2}\frac{\xi(T_2, Y_2)}{(T_k + \overline{T}_k)}|C_j^{5_i}|^2 d_{ijk} \\ & + \frac{1}{2}\sum_i \frac{\xi(T_2, Y_2)}{(S+\overline{S})^{1/2}(T_k + \overline{T}_k)^{1/2}}|C^{5_i 5_j}|^2 d_{ijk} \\ & + \frac{1}{2}\sum_{i\neq 2}\frac{\xi(T_2, Y_2)}{(T_j + \overline{T}_j)^{1/2}(T_k + \overline{T}_k)^{1/2}}|C^{95_i}|^2 d_{ijk} \end{aligned} \tag{29}$$

and

$$K(S,\overline{S},T_i,\overline{T}_i)_{unseq.} = -\ln(S+\overline{S}) - \sum_{i=1}^{3}\ln(T_i+\overline{T}_i) + \sum_{i=1}^{3}\frac{|C_i^9|^2}{(T_i+\overline{T}_i)} + \frac{|C_2^{5_2}|^2}{(S+\overline{S})} + \frac{1}{2}\sum_{i}^{3}\frac{|C_k^{5_2}|^2}{(T_i+\overline{T}_i)}d_{ik} + \frac{|C^{95_2}|^2}{(T_1+\overline{T}_1)^{1/2}(T_3+\overline{T}_3)^{1/2}} \quad (30)$$

where

$$\xi(T_2,Y_2) = exp\left[\frac{1}{6}\left(1-e^{-(T_2+\overline{T}_2)\lambda_I/4}\right)\left\{Y_2+\overline{Y}_2-\delta_{GS}\ln(T_2+\overline{T}_2)\right\}^2\right]. \quad (31)$$

The full Kahler potential is the sum of Eq. (29) and Eq. (30).

For the values of the parameters in our model, $\xi \approx 1$. If this were not the case canonical normalisation of the Kahler potential would produce a superpotential different from Eq. (5).

Having specified our Kahler potential we follow the procedure in [9] and calculate the new soft terms. They are parametrised by Goldstino angles θ and ϕ and Goldstino parameters Θ_i (where $\sum_{i=1}^{3}\Theta_i^2 = 1$).

We now present the full expressions for the relavant soft masses, $\tilde{m}$, $A_{C_3^{5_1}C^{5_15_2}C^{5_15_2}}$ and $A_{C_3^{5_2}C^{5_15_2}C^{5_15_2}}$, up to $\mathcal{O}\left(\frac{1}{T_2+\overline{T}_2}\right)$

$$\begin{aligned} m_Q^2 = m_{C_3^{5_2}}^2 &= m_{3/2}^2 - 3m_{3/2}^2\Theta_1^2\cos^2\theta\sin^2\phi \\ m_0^2 = m_{C^{5_15_2}}^2 &= \tilde{m}^2 - \frac{3}{2}m_{3/2}^2\left(\sin^2\theta + \Theta_3^2\,\cos^2\theta\,\sin^2\phi\right) \\ m_\phi^2 = m_{C_3^{5_1}}^2 &= \tilde{m}^2 - \frac{3}{k}m_{3/2}^2\Theta_2^2\cos^2\theta\sin^2\phi. \end{aligned} \quad (32)$$

$$
\begin{aligned}
\tilde{m}^2 = m_{3/2}^2 \Bigg[& 1 - \cos^2\theta \cos^2\phi \left(1 - e^{-\lambda_I (T_2+\overline{T}_2)/4}\right) \\
& - \frac{\cos^2\theta \sin^2\phi \, \Theta_2^2 \, \delta_{GS}}{k} \left(1 - e^{-\lambda_I (T_2+\overline{T}_2)/4}\right) \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\} \\
& + \frac{\cos^2\theta \sin^2\phi \, \Theta_2^2 \, e^{-\lambda_I (T_2+\overline{T}_2)/4}}{32k} \lambda_I^2 (T_2 + \overline{T}_2)^2 \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}^2 \\
& - \frac{\lambda_I \cos^2\theta \cos\phi \sin\phi \left(\Theta_2 \, e^{i(\alpha_2 - \alpha_{Y_2})} + \Theta_2^\dagger \, e^{-i(\alpha_2 - \alpha_{Y_2})}\right) e^{-\lambda_I (T_2+\overline{T}_2)/4}}{32\sqrt{k}} \\
& \times \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\} \left(8(T_2 + \overline{T}_2) + \lambda_I \delta_{GS} \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}\right) \Bigg] .
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
A_\lambda = A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} = -\sqrt{3} m_{3/2} \cos\theta \Big[& \sin\phi \, \Theta_1 \, e^{i\alpha_1} \\
& + \sin\phi \frac{\Theta_2 \, e^{i\alpha_2}}{8\sqrt{k}} \, e^{-\lambda_I (T_2+\overline{T}_2)/4} \, \lambda_I (T_2 + \overline{T}_2) \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}^2 \\
& - \cos\phi \, e^{i\alpha_{Y_2}} \, e^{-\lambda_I (T_2+\overline{T}_2)/4} \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\} \Big]
\end{aligned}
\tag{34}
$$

$$
\begin{aligned}
A_Q = A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} = -\sqrt{3} m_{3/2} \cos\theta \Bigg[& \sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{\sqrt{k}} \\
& + \sin\phi \frac{\Theta_2 \, e^{i\alpha_2}}{12\sqrt{k}} \, e^{-\lambda_I (T_2+\overline{T}_2)/4} \, \lambda_I (T_2 + \overline{T}_2) \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}^2 \\
& - \cos\phi \, \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2e^{-\lambda_I (T_2+\overline{T}_2)/4}\right) \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\} \Bigg]
\end{aligned}
\tag{35}
$$

For our value of λ_I it is interesting to note that, to a very good approximation, $\tilde{m} = m_{3/2}$, consistent with [3], so the effects of the sequestering are not felt by the soft masses. Also the exponentials vanish from A_λ so it is not the sequestering that breaks the sum rule in Eq. (18). This should not come as a surprise because R_2 is close to the Planck length, as we saw in Eq. (10), so there is next to no spatial separation between Y_2 and $D5_1$. The sum rule is broken by the Kahler potential for the twisted moduli in Eq. (28). If it was logarithmic, as all the other moduli are, then the sum rule would hold, but the fact it is quadratic breaks the sum rule.

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